§9.5: Two ~~Population~~ "Sample" Variances "F-Test"

We prefer to use "pooled variance two sample t-test" rather than the normal "two sample t-test" because it has larger #deg. of freedom so smaller p-value...

Q: When can you assume $Var[X] = Var[Y]$?

Def: If $\left.\begin{array}{l} X_1 \sim \chi^2(\nu_1) \\ X_2 \sim \chi^2(\nu_2) \end{array}\right\}$ then $\dfrac{X_1/\nu_1}{X_2/\nu_2} \sim F(\nu_1, \nu_2)$

"Chi-Squared"

"F-distribution with ν_1 numerator deg. of freedom ν_2 denominator deg. of freedom."

[Recall from §7.4 that if X is sampled n times then $\dfrac{(n-1)S_X^2}{\sigma_X^2} \sim \chi^2(n-1)$.

Thus if X is sampled n times & Y sampled m times

Thm: If X & Y are sampled n & m times then

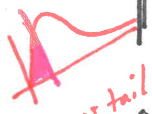
$$F = \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim F(n-1, m-1)$$

Hypothesis Test: $H_0: \sigma_X^2 = \sigma_Y^2$

Test Statistic:

$$F = \frac{S_x^2}{S_y^2} \sim F(n-1, m-1)$$

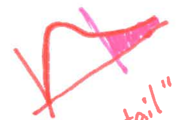
p-value: (one-tailed)



$$pf\left(S_x^2/S_y^2,\ n-1,\ m-1\right)$$

"lower tail"

– or –



$$1 - pf\left(S_x^2/S_y^2,\ n-1,\ m-1\right)$$

"upper tail"

(one of these will be $> 1/2$ and the other is $< 1/2$. p-value is the one $< 1/2$.)

Note: You can also test $H_0: \sigma_Y^2 = a\,\sigma_X^2$

[In this case the test statistic is

$$F = \frac{S_x^2/\sigma_X^2}{S_y^2/\sigma_Y^2} = \frac{S_x^2/\sigma_X^2}{S_y^2/a\,\sigma_X^2} = a \cdot \frac{S_x^2}{S_y^2}$$